

Here are some questions like those that may appear on the test on the prerequisites. As you can see, they're pretty easy. If you are in command of the prerequisites, you'll do fine.

1. (a) Compute the first 3 terms of the Taylor series for $f(x) = \ln(2 + \sin(x))$ about $x_0 = \pi$.
- (b) Let $f(x, y, z) = xye^{z^2}$. Compute $\partial^2 f / \partial x \partial z$.
- (c) Compute $\int_0^1 \int_0^1 ye^{xy} dx dy$.
- (d) Use your knowledge of calculus to give the Taylor series of $f(x) = \sin(x^3)$, $f(x) = \ln(1 + x^3)$, and $f(x) = (x - 1)^4$ about $x_0 = 0$.
- (e) Evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sin(\pi(x^2 + y^2)) dy dx$$

2. Let

$$x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, y = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \text{ and } z = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

- (a) Find a 2×2 matrix A such that

$$Ax = y \text{ and } Ay = z.$$

Is this matrix unique?

- (b) Find a matrix B such that

$$Bx = y \text{ and } By = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Is B unique? Compute the eigenvalues and eigenvectors of B .

3. Let

$$A = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \text{ and } b = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

- (a) Compute the eigenvalues and corresponding eigenvectors of A .
 - (b) Solve $Ax = b$ by Gaussian elimination.
 - (c) Compute $b^T A b$, $b^T A^T b$, $A b b^T$, and $b^T A^T A b$.
4. Write a function or subroutine that takes as input two $N \times N$ matrices A and B and returns AB^T . You may use any of FORTRAN, MATLAB, or C, but your routine must be correct.
5. Write a main program that calls the function or subroutine that you wrote in the previous problem to compute $x^T AB^T y$ where

$$A = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}, x = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ and } y = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

6. Write a function or subroutine that compares e^x to the first N terms of its Taylor series about 0 for $N = 1, 2, \dots, 100$. Your function or subroutine should take x as the input and produce an array of 100 real numbers as output.
7. Write a program that approximates $\int_0^1 e^x \cos(x) dx$ with the trapezoid rule and Simpson's rule using 101 points. Your program should print the result on the screen (or standard output).